Exercises for October 31, 2009

1. One bottle and one glass weigh as much as one pitcher. One bottle weighs as much as one glass and one saucer. Two pitchers weigh as much as three saucers. How many glasses will balance one bottle?
2. John said that there were more than six guests at his birthday party. His sister said that there were more than five guests. How many guests did come to the party if it is known that exactly one of these two statements is true, and the other is false?
3. One has 19 metal balls which weigh 1 gram, 2 grams, ..., 19 grams. It is known that 9 of the balls are made of steel, 9 of brass and 1 of gold. The total mass of all brass balls is 90 grams less than the total mass of all steel balls. Find the mass of the golden ball.
4. Position five identical wooden cubes in space in such a way that any two of them would touch each other (over a part of their face, not only by an edge or a corner).
5. If Sophie walks to the beach and returns back home by bus, she spends an hour and a half on her road. If she takes the bus both ways, she spends only thirty minutes. How much time would it take for Sophie to walk to the beach and back?
6. A 6 ounce glass has 4 ounces of juice in it. Another 7 ounce glass has 6 ounces of juice. How can one end up with 5 ounces of juice in each glass using only these two glasses and a small 3 ounce coffee cup?
7. Can a chess knight travel from the lower left corner of the chess board to its upper right corner, visiting each square exactly once? Either give an example of such a route or explain why it is not possible.

Homework

1. Three mice are trying to share between themselves three piece of cheese which weigh 5 grams, 8 grams and 11 grams. A hamster is trying to help them. He is allowed to cut off and eat 1 gram of cheese from any two pieces of cheese at a time. Can the hamster leave three equal pieces of cheese after several such operations?   (Hamsters *can* eat cheese, but mostly they don't like it too much.)
2. Can you place several security cameras around a diamond is such a way that a thief could not approach unnoticed neither the diamond nor any of the cameras? Each of the cameras points in one direction, does not move, and shows the line exactly 100 yards ahead of it.
3. Can you place a number of checkers on a 8x8 chess board (at most one checker in one square), in such a way that each of the vertical lines has the same number of checkers but any two horizontal lines have different numbers of checkers? If your answer is positive, draw a picture, if negative - explain why this is not possible.