# Problems for January 9, 2010

## Even and Odd

1. Will the sum of two even numbers be odd or even? What about three odd numbers?
2. Prove that, given any three integers, one can choose two of them which have even sum.
3. Is it possible that the sum of three numbers is even but their product is odd?
4. Is it possible to pay exactly 20 cents (without change returned) using seven coins (pennies, nickels and dimes)? What about 20 cents with seven coins, but using pennies and nickels only? What about 25 cents with eight coins, but again using pennies and nickels only?
5. Andrew has bought at a store 20 notebooks, several pencils at the price $3.20 and several erasers at the price $2.20. He noted that the pre-tax total showed $55.65 and said that one of the items did not scan correctly. The mistake was fixed. How did he know?
6. An 8x8 chess board was stained with paint. Is it possible that the number of squares not stained minus the number of square which *were* stained, gives 17?
7. A chess knight has left the square A1, made several moves and returned back to the same square. Prove that it made an even number of moves.
8. Several gears are joined in a circular fashion. Can they rotate if there are (a) twelve, (b) thirteen, gears total?
9. Seven positive integers are written in a circle. Is it true that there will always be two standing next to each other, which have even sum?
10. The following equality is written on the board: 1*2*3*4*5*6*7*8*9=20 (but instead of each * there is either a + or a -). Show that the equality cannot be correct.
11. A board of size 25x25 has 25 checkers, and the whole configuration is symmetric with respect to a diagonal. Show that one of the checkers must be standing on the diagonal.
12. Four numbers are written on the board: 0, 0, 0 and 1. You are allowed to make several moves, each of the moves is to add 1 to any two of these numbers. Can you make all four numbers equal after several moves?
13. A Hero is fighting a Dragon with three heads and three tails. With one blow the Hero can cut off either one head, or one tail, or two heads, or two tails. But - if one tail is cut off then two tails grow in its place; if two tails - a head grows instead; if one head is cut off - a new head grows in its place; and if two heads are cut off then nothing grows in their place. Can the Hero cut off all heads and all tails of the Dragon?
14. A fox and two bear cubs are trying to share 100 candies. The fox is splitting all candies into three bunches, and each will take one of the bunches randomly. The fox knows that if the bear cubs get bunches with different numbers of candies they will ask her to make these equal, which she will do by taking the excess for herself.

After that each will eat the candies. Can you make an arrangement which guarantees that the fox will eat exactly 80 candies? How about 65 candies?