Problems for October 17, 2009

1. An elevator of a 20-floor building has just two buttons. One of the buttons takes the elevator 13 floors up, the other 8 floors down. Can you use this elevator to go from the 13th floor to the 8th floor?
2. Put 10 chairs next to the walls of a square shaped room in such a way that the same number of chairs will be standing along each wall.
3. Prove that in any group of 5 people there will be two who have the same number of friends in this group.
4. A supermarket gets 25 boxes with three types of apples (Fuji, Gala and Washington). Each box has only one type of apples in it. The manager wants 9 boxes of apples of the same type but the store associate says that he can't find that many. Do you think he is wrong?
5. 20 hikers went on a hike. The eldest is 35 years old and the youngest is (a) 16 years old (b) 17 years old. Is it true that there will be two hikers of the same age?
6. A school has 400 students. Prove that at least two of them will have their birthday on the same day and month (even if they were born in different years).
7. The meetings of a (very large) math circle take place in 9 classrooms. Among other students, 19 come from the same school. (a) Show that with any seating arrangement there will be at least one room with at least three such students; (b) is it true that some room will have *exactly* three such students?

Homework

1. Prove that no line can intersect all three sides of a triangle.
2. What is the maximal number of chess kings that one can place of a chess board, in such a way that no king attacks another?
3. Each face of a cube is painted either black or white. Prove that there will be two faces which share an edge and are painted in the same color.