Math2E - Practice Final

December 4, 2007

1. Use the transformation $u = x - y, v = x + y$ to evaluate $\int\int_R (x-y)/(x+y)dA$, where R is the square with vertices $(0,2),(1,1),(2,2),$ and $(1,3)$.

2. Evaluate $\int_C xydx + ydy$, C is the sine curve $y = \sin x, 0 \leq x \leq \pi/2$.

3. $\mathbf{F}(x,y,z) = e^y\mathbf{i} + (xe^y + e^z)\mathbf{j} + ye^z\mathbf{k}$,
   (a): Show that $\mathbf{F}$ is conservative,
   (b): Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the line segment from $(0,2,0)$ to (4,0,3).

4. Find the area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices $(0,0),(1,0)$ and (1,2).

5. Evaluate $\int\int_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = x^2\mathbf{i} + xy\mathbf{j} + z\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$ with upward orientation.

6. Evaluate $\int\int_S \text{curl}\mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = x^2yz\mathbf{i} + yz^2\mathbf{j} + z^3e^{xy}\mathbf{k}$, S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$, and S is oriented upward.

7. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z) = xy\mathbf{i} + yz\mathbf{j} + zxbfk$ and C us the triangle with vertices $(1,0,0),(0,1,0)$, and $(0,0,1)$, oriented counterclockwise as viewed above.

8. Evaluate $\int\int_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,,z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.