Math107L - Project 3

Due: June 15, 2007

June 5, 2007

$$\begin{array}{ll} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4 & 0 < x < 1, \quad 0 < y < 2; \\ u(x,0) = x^2, \quad u(x,2) = (x-2)^2, & 0 \le x \le 1; \\ u(0,y) = y^2, \quad u(1,y) = (y-1)^2, & 0 \le y \le 2; \end{array} \tag{1}$$

1. (30 points) Use Finite Difference method to solve (1) with the step size $\Delta x = \Delta y = 0.5$, compare the results to the actual solution $u(x,y) = (x-y)^2$.

$$\begin{array}{ll} \frac{\partial u}{\partial t} - \frac{4}{\pi^2}\frac{\partial^2 u}{\partial x^2} = 0 & 0 < x < 4, \quad t > 0; \\ u(0,t) = u(4,t) = 0, & t > 0; \\ u(x,0) = \sin(\pi x/4)(1 + 2\cos(\pi x/4), & 0 \le x \le 4; \end{array} \tag{2}$$

2. (30 points) Use Forward Difference method to solve (2) at $t = 0.4$ with the step size $\Delta x = 0.2.\Delta t = 0.04$, compare your results at $t = 0.4$ to the actual solution $u(x,t) = e^{-t}\sin(\pi x/2) + e^{-t/4}\sin(\pi x/4)$.

$$y'' = 2y^3 - 6y - 2x^3, \quad 1 \le x \le 2, \quad y(1) = 2, y(2) = 5/2, \tag{3}$$

3. (40 points) Use NonLinear Finite Difference method with $TOL = 10^{-4}$ to solve (3) with the step size $h = 0.1$, compare your results to the actual solution $y(x) = x^{-1} + \ln x$.