$\boxed{\textbf{Math2E - Practice Final}}$

June 5, 2009

1. Evaluate $\int_C xydx + ydy$, C is the sine curve $y = \sin x, 0 \leq x \leq \pi/2$.
   Answer: $\frac{3}{2}$. Hint: $\int x \sin x dx = -x \cos x + \int \cos x dx$

2. $\mathbf{F}(x, y, z) = e^y\mathbf{i} + (xe^y + e^z)\mathbf{j} + ye^z\mathbf{k}$,
(a): Show that $\mathbf{F}$ is conservative,
(b): Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the line segment from $(0, 2, 0)$ to (4,0,3).
Answer: (a) $\mathbf{F}$ is conservative, (b) 2.

3. Evaluate $\int_C x^2 y dx + \ln\sqrt{1+y^2}dy$, where C is the triangle from $(0,0)$ to $(2,2)$ to $(0,2)$ to $(0,0)$ with counterclockwise orientation.
Answer: $-\frac{4}{3}$.

4. Evaluate $\int\int_S \mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F}(x, y, z) = x^2\mathbf{i} + xy\mathbf{j} + z\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$ with upward orientation.
Answer: $\frac{\pi}{2}$.

5. Evaluate $\int\int_{\partial Q} \mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F} =< x^2 - y^2 z, x \sin z, 4y^2 >$, Q is bounded by $4x + 2y - z = 4 \;\; (z \leq 0)$ and the coordinate planes.
Answer: $-\frac{2}{3}$.

6. Evaluate $\int\int_S \text{curl}\mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + yz^2\mathbf{j} + z^3e^{xy}\mathbf{k}$, S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$, and S is oriented upward.
   Answer: -4π.

7. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$ and C is the triangle with vertices $(1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$, oriented counterclockwise as viewed above.
Answer: $-\frac{1}{2}$.

8. Evaluate $\int\int_S \mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F}(x, y, , z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.
Answer: 11π.