Math3D - Practice Final

March 12, 2008

1. Find the solution of the given initial-value problem
   $\frac{dy}{dt} + ty = 1 + t, \quad y(0) = 3/2.$

2. Solve the given initial-value problem
$2t\cos y + 3t^2 y + (t^3 - t^2 \sin y - y)\frac{dy}{dt} = 0, \quad y(0) = 2.$

3. Solve the following initial-value problem
$4\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + y = 0, \quad y(0) = 0, \quad y'(0) = 3.$

4. Find the general solution of the following equation
$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = te^{2t}$.
(Hint: Method of variation of parameters or judicious guessing)

5. Find a particular solution of the following equation
$y'' - 2y' + 5y = 2(\cos^2 t)e^t$
(Hint: Method of judicious guessing)

6. Solve the following initial-value problem
   $y'' + (t^2 + 2t + 1)y' - (4 + 4t)y = 0;\quad y(-1) = 0,\quad y'(-1) = 1.$
   (Hint: Series solution)

f(t)	1	$e^{\alpha t}$	$\cos \omega t$	$\sin \omega t$	t^n	$t^n e^{\alpha t}$	$t \cos \omega t$	$t \sin \omega t$
$\mathcal{L}\{f(t)\}$	$\frac{1}{s}$	$\frac{1}{s-\alpha}$	$\frac{s}{s^2+\omega^2}$	$\frac{\omega}{s^2+\omega^2}$	$\frac{n!}{s^{n+1}}$	$\frac{n!}{(s-\alpha)^{n+1}}$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	$\frac{2\omega s}{(s^2+\omega^2)^2}$

7. Solve the following initial-value problem by method of Laplace transforms
$y'' + y' + y = 1 + e^{-t}; \quad y(0) = 3, \quad y'(0) = -5.$

8. Solve the given initial-value problem

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1}$$