Math2B - Practice Final

March 10, 2009

1. $\int_0^1 (1-x)^9 dx =$
   (A) $\frac{-1}{9}$ (B) $\frac{1}{10}$ (C) $\frac{-1}{10}$ (D) $\frac{1}{9}$. B

2. $f(x) = x^3 + 4x - 1$, the derivative of $f^{-1}(x)$ at $x = -1$ is C
   (A) $\frac{1}{7}$ (B) 7 (C) $\frac{1}{4}$ (D) 4.

3. The derivative of $y = \ln(x^2 e^x)$ is
   (A) $\frac{2}{x} + 1$ (B) $\frac{1}{x^2 e^x}$ (C) $\frac{2}{x}$ (D) $\frac{1}{x} + 1$. A

4. $\lim_{x \to 0} \frac{\cos x}{x^2} =$
   (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D)∞. D

5. $\lim_{x \to 0^+} x^x =$
   (A) 0 (B) 1 (C) -1 (D) ∞ B

6. $\int \frac{x+9}{x^2+9} =$ C
   (A) $\ln\sqrt{x^2+9} + \arctan(\frac{x}{3})$ (B) $\ln(x^2+9) + \arctan(\frac{x}{3})$ (C) $\ln\sqrt{x^2+9} + 3\arctan(\frac{x}{3})$
   (D) $\ln(x^2+9) + 3\arctan(\frac{x}{3})$.

7. $\tan(\arcsin \frac{1}{2}) =$
   (A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$. B

8. The length of $y = \frac{4}{3}\sqrt{x^3}, 0 \le x \le 1$ is
   (A) $\frac{2(\sqrt{125}-1)}{3}$ (B) $\frac{\sqrt{125}-1}{6}$ (C) $\frac{2(\sqrt{5}-1)}{3}$ (D) $\frac{\sqrt{5}-1}{6}$ B

9. $\int_2^5 \frac{1}{\sqrt{x-2}} dx =$
   (A) Divergent (B) $2\sqrt{3}$ (C) $-2\sqrt{3}$ (D) $\sqrt{3}$. B

10. The derivative of 3^{2x} is
    (A) 3^{2x} (B) $2\ln 3 3^{2x}$ (C) $\frac{1}{2}\ln 3 3^{2x}$ (C) $\ln 3 3^{2x}$ B.

15: (a) $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx \xrightarrow{u=e^x,\ du=e^x dx} \int \frac{u}{u^2+3u+2}du = \int\left(\frac{2}{u+2}-\frac{1}{u+1}\right)du$

$= 2\ln|e^x+2| - \ln|e^x+1| + C$

(b): $\int_1^2 x^4(\ln x)^2dx = \int_1^2 (\ln x)^2 \cdot d(\frac{1}{5}x^5) = \frac{1}{5}x^5\cdot(\ln x)^2\Big|_1^2 - \int_1^2 \frac{1}{5}x^5\, 2\ln x\cdot\frac{1}{x}dx$

$= \frac{32}{5}(\ln 2)^2 - \frac{2}{5}\int_1^2 x^4\ln x\,dx = \frac{32}{5}(\ln 2)^2 - \frac{2}{5}\left[\frac{1}{5}x^5\ln x\Big|_1^2 - \frac{1}{5}\cdot\frac{1}{5}x^5\Big|_1^2\right]$

$= \frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln 2 + \frac{62}{125}$

11. (a) Find the are of the region bounded by $x+y=0, x=y^2+3y$.
(b): Find the volume of the solid obtained by the rotating $y=x^2+1, y=9-x^2$ about $y=-1$.

12. (a) Evaluate $\int_1^e 4t^2\ln t dt$
(b) Evaluate $\int_{\sqrt{2}}^2 \frac{1}{x^3\sqrt{(x^2-1)}}dx$.

13. (a) Evaluate $\int \cos^5 x \sin^4 x dx$. $\xrightarrow{u=\sin x,\ du=\cos x dx} \int (1-u^2)^2 \cdot u^4 du = \frac{1}{9}\sin^9 x - \frac{2}{7}\sin^7 x + \frac{1}{5}\sin^5 x + C$
(b) Evaulate $\int_0^{\pi/3}\tan^5 x \sec^4 x dx$. $\xrightarrow{u=\tan x,\ du=\sec^2 x dx} \int_0^{\sqrt{3}} u^5\cdot(1+u^2)du = \frac{117}{8}$

14. (a) Evaluate $\int_2^3 \frac{1}{x^2-1}dx$. $= \int_2^3 \frac{1}{2}\left(\frac{-1}{x+1}+\frac{1}{x-1}\right) = \frac{1}{2}\ln 3$
(b) Evalute the improper integral $\int_0^1 \ln x dx$. $= \lim_{R\to 0^+}\int_R^1 \ln x dx = \lim_{R\to 0^+}\left(x\cdot\ln x\Big|_R^1 - \int_R^1 1 dx\right)$

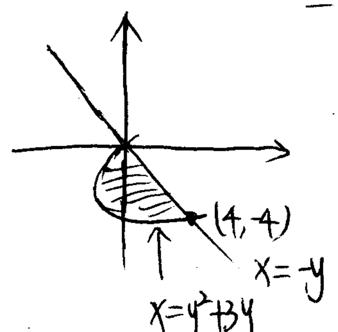
$= -1$

15. (a) Evaluate $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$.
(b) Evalute $\int_1^2 x^4(\ln x)^2dx$.

11: (a):

$A = \int_{-4}^{0}\left[-y-(y^2+3y)\right]dy$

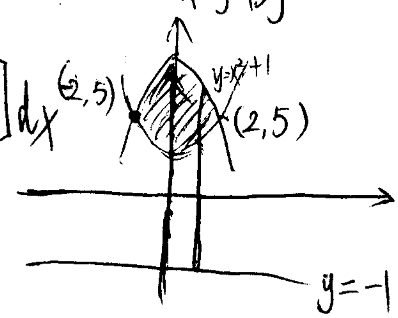
$= \frac{32}{3}$



(b):

$V = \int_{-2}^{2} \pi\cdot\left[(x^2+2)^2 + (10-x^2)^2\right]dx$

$= 276\pi$



12: (a): $\int_1^e 4t^2\ln t\,dt = \int_1^e \ln t\,d(\frac{4}{3}t^3) = \frac{4}{3}t^3\cdot\ln t\Big|_1^e - \int_1^e \frac{1}{t}\cdot\frac{4}{3}t^3dt$

$= \frac{4}{3} - \frac{4}{9}(e^3-1) = \frac{16}{9} - \frac{4}{9}e^3$

(b) $\int_{\sqrt{2}}^2 \frac{1}{x^3\sqrt{(x^2-1)}}dx \xrightarrow[x=\sqrt{2},\ \theta=\frac{\pi}{4}]{x=\sec\theta,\ x=2,\ \theta=\frac{\pi}{3}} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sec^3\theta\cdot\tan\theta}\sec\theta\cdot\tan\theta\,d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2\theta d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\cos 2\theta}{2}d\theta = \frac{\pi}{24} + \frac{1}{4} - \frac{\sqrt{3}}{8}$