Calabi-Yau moduli space

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The study of moduli space of polarized Calabi-Yau manifolds (CY moduli) is related to several branches of mathematics:

1. differential geometry
2. algebraic geometry
3. mathematical physics (in particular string theory)
4. and others

The differential geometry of CY moduli

We assume that $\mathcal{M}$ is the CY moduli of some Calabi-Yau threefolds with dimension m. Assume that ω is the Weil-Petersson metric of $\mathcal{M}$. In [5], we defined

$$\omega' = (m+3)\omega + \mathrm{Ric}(\omega),$$

and we call ω' the Hodge metric. The curvature of ω' has good properties [4, 5].

In 1993, physicists Bershadsky, Cecotti, Ooguri, and Vafa proposed the so-called BCOV conjecture. The BCOV conjecture states that the computation of the Gromov-Witten instantons on the Calabi-Yau 3 fold would be the same as the BCOV torsion of the mirror Calabi-Yau threefold.

1. Symplectic side of the BCOV conjecture: proved by Zinger [6]
2. Complex Geometric side of the BCOV conjecture: proved by Fang-Lu-Yashikawa [2]

In general, the CY moduli space is not complete. In the following [1], we proved

Theorem

The integration of the Chern-Weil forms of the Weil-Petersson metric is a rational number.

Special Kähler manfold is dual to CY moduli. In [3], we proved the Freed Conjecture:

Theorem

Any special Kähler manifold is incomplete unless it is flat.

References.

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