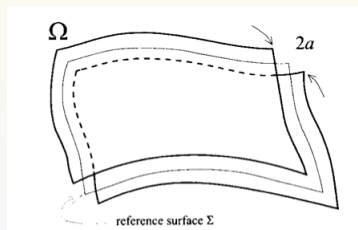


# Quantum Layer Conjecture

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The concept of *quantum layer* was introduced in **mesoscopic physics**. A quantum layer is a kind of complete non-compact manifold with boundary.



**Figure** : Picture of a quantum layer. By Carron, Exner, Krejčířík [1]

## Definition

Suppose that  $\Sigma$  is a complete non-compact embedded surface in  $\mathbb{R}^3$ . We assume that the second fundamental form of  $\Sigma$  goes to zero at infinity. Let  $a$  be a small positive number. The quantum layer  $\Omega$  is defined as the set of points in  $\mathbb{R}^3$  such that the distance to  $\Sigma$  is no more than  $a$ .

Duclos, Exner, and Krejčířík [2] and Carron, Exner, and Krejčířík [1] proved

### Theorem

*Let  $\Omega$  be a quantum layer and let  $K$  be the Gauss curvature. If*

1

$$\int_{\Sigma} |K| d\Sigma < \infty, \text{ and}$$

2

$$\int_{\Sigma} K d\Sigma \leq 0$$

*Then the ground state exists.*

Base on their work, we make the following

### Conjecture (Quantum Layer Conjecture)

*The ground state also exists if we remove Condition 2 in the above theorem.*

The conjecture is proved by Lu and Rowlett [6].

# References.

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