What is Differential Geometry?

Zhiqin Lu

UCI, Recruitment Day

April 3, 2009

Triple integrals

Compute

$$\iiint_W x dx dy dz,$$

where W is the region bounded by the planes $x = 0, y = 0$, and $z = 2$, and the surface $z = x^2 + y^2$ and lying in the quadrant $x \geq 0, y \geq 0$.

Triple integrals

Compute

$$\iiint_W x dx dy dz,$$

where W is the region bounded by the planes $x = 0, y = 0$, and $z = 2$, and the surface $z = x^2 + y^2$ and lying in the quadrant $x \geq 0, y \geq 0$.

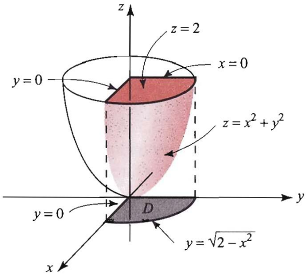


Figure: From *Vector Calculus*, Marsden & Tromba

How to compute integrations over an n-dimensional object?

An example

Zeros of a quintic polynomial:

$$Z_0^5 + Z_1^5 + Z_2^5 + Z_3^5 + Z_4^5 + 10Z_0Z_1Z_2Z_3Z_4 = 0$$

in $\mathbb{C}^5$.

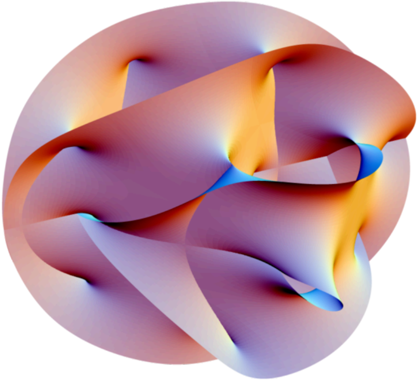


Figure: From Wikipedia, the intersection of the quintic Calabi-Yau threefold to our three dimensional space

How to study high dimensional geometric object?

- Use Partial Differential Equations;

How to study high dimensional geometric object?

- Use Partial Differential Equations;
- Use Linear Algebra

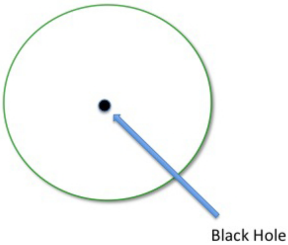
How to study high dimensional geometric object?

- Use Partial Differential Equations;
- Use Linear Algebra
- Use Abstract Algebra

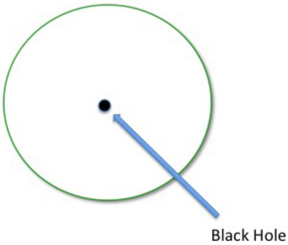
How to study high dimensional geometric object?

- Use Partial Differential Equations;
- Use Linear Algebra
- Use Abstract Algebra
- Use the results in all other math/physics fields.

A simple example



A simple example



$$\frac{1}{2\pi}\oint \frac{xdy - ydx}{x^2 + y^2} = 1.$$

A non-trivial example

How many holes in the quintic Calabi-Yau manifold?

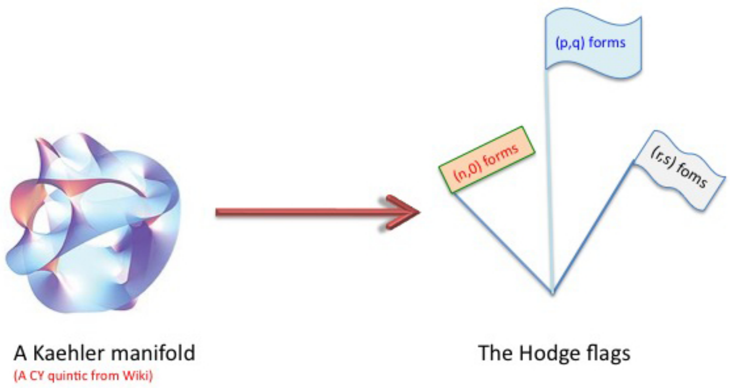
In 1977, S. T. Yau was able to solve the following PDE

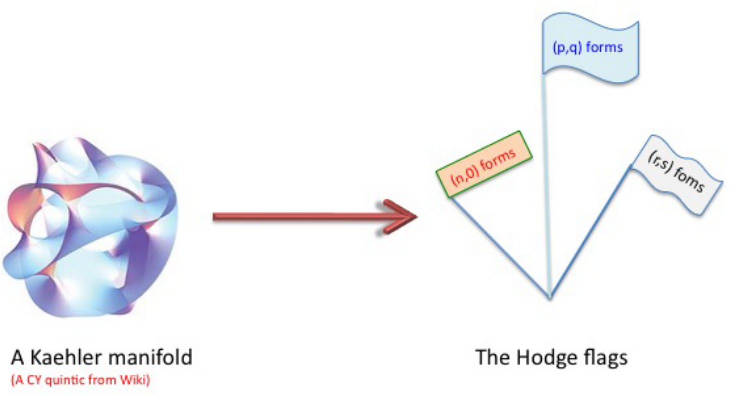
$$\det\begin{pmatrix} g_{1,\bar 1}+\frac{\partial^2 u}{\partial z_1\partial \bar z_1} & g_{1,\bar 2}+\frac{\partial^2 u}{\partial z_1\partial \bar z_2} & g_{1,\bar 3}+\frac{\partial^2 u}{\partial z_1\partial \bar z_3}\\ g_{2,\bar 1}+\frac{\partial^2 u}{\partial z_2\partial \bar z_1} & g_{2,\bar 2}+\frac{\partial^2 u}{\partial z_2\partial \bar z_2} & g_{2,\bar 3}+\frac{\partial^2 u}{\partial z_2\partial \bar z_3}\\ g_{3,\bar 1}+\frac{\partial^2 u}{\partial z_3\partial \bar z_1} & g_{3,\bar 2}+\frac{\partial^2 u}{\partial z_3\partial \bar z_2} & g_{3,\bar 3}+\frac{\partial^2 u}{\partial z_3\partial \bar z_3}\end{pmatrix}$$

$$= e^F \det\begin{pmatrix} g_{1,\bar 1} & g_{1,\bar 2} & g_{1,\bar 3}\\ g_{2,\bar 1} & g_{2,\bar 2} & g_{2,\bar 3}\\ g_{3,\bar 1} & g_{3,\bar 2} & g_{3,\bar 3}\end{pmatrix},$$

where $g_{i\bar j}$ and F are given functions. After that, we are able to tell the topological properties of the manifold.

Linear algebra





Hodge theory!

Conclusion:

Differential Geometry is not a separate math field, it brought different fields like PDE, algebraic geometry, algebraic topology, Lie group theory, functional analysis, and many others together.

UCI's Geometry / Topology Group

UCI's Geometry / Topology Group

- Peter Li,

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;
- Chuu-Lian Terng,

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;
- Chuu-Lian Terng, Differential Geometry, Integrable systems

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;
- Chuu-Lian Terng, Differential Geometry, Integrable systems
- Ronald J. Stern,

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;
- Chuu-Lian Terng, Differential Geometry, Integrable systems
- Ronald J. Stern,Smooth 4-manifolds, symplectic contact topology and geometry, knot theory

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;
- Chuu-Lian Terng, Differential Geometry, Integrable systems
- Ronald J. Stern,Smooth 4-manifolds, symplectic contact topology and geometry, knot theory
- Zhiqin Lu,

UCI's Geometry / Topology Group

- Peter Li, Geometric PDE, Analysis on manifolds;
- Chuu-Lian Terng, Differential Geometry, Integrable systems
- Ronald J. Stern,Smooth 4-manifolds, symplectic contact topology and geometry, knot theory
- Zhiqin Lu, complex geometry, Mirror symmetry.

http://math.uci.edu/~zlu/geom-topo-group/index.php

Members of the Geometry & Topology Group at UCI work in many different fields and have expertise in a diverse set of techniques. We have lively and well-attended seminars, and one of our key goals is the cross-pollination of ideas between geometry and topology.

Our faculty consists of active researchers in many areas of geometry and low-dimensional topology including geometric PDE, differential geometry, integrable systems, mirror symmetry, smooth 4-manifolds, symplectic and contact topology and geometry, and knot theory and its invariants.

Our faculty is highly-regarded. All have NSF Grants and one of its members, Peter Li, was elected to the American Academy of Arts and Sciences. The Geometry/Topology Group at UCI has a long-standing commitment to excellence in graduate and postdoctoral training: we have produced some outstanding graduate students, and we have been fortunate to have recruited and mentored exceptional postdoctoral fellows.

Welcome to UCI!